RSAC 2025 Conference

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SESSION ID: CRYP-M05A

How (Not) to Hash into Class Groups of Imaginary Quadratic Fields?

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  - Verifiable delay functions [BBBF18],
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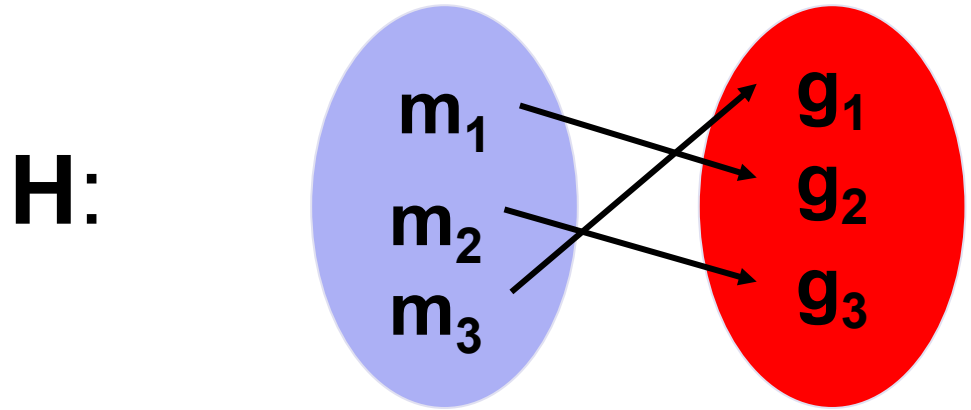
Hashing to (class) groups?! Why care?

- **Many applications** need a secure hash function to class groups!
  - Verifiable delay functions [BBBF18],
  - Zero-knowledge proofs [BFS20],
  - Timed commitments [TCLM21], and more!
- But, it's often **tricky mapping some message** *m* to a group **G**.



Why are class groups great in crypto?

RSA groups

Class groups

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- For a group $\mathbf{G}=\mathbf{Z}^*_N$:
  - G's order is unknown ***only if certain conditions are met!***

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- *Karatsuba* multiplication algorithm for efficient group operations.

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- **Transparent setup!** (No MPC!)
- Binary quadratic forms (see later)
- For 128-bit security $\log_2\mathbf{\Delta}\approx 6800$.
- *Shanks'* NUCOMP and NUDPL algorithms are used for efficient class group operations.

Our contributions

https://github.com/seresistvanandras/hashingToClassGroups

Our contributions

- We identify two families of **broken** hash functions to class groups.
  - SageMath's hash function to class groups is insecure.
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- We identify two families of **secure** hash functions to class groups.
  - An algebraic hash function.
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- Open-source **implementation** and performance evaluation.

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Agenda

- **Preliminaries** on class groups of imaginary quadratic fields
  - What is a binary quadratic form (a,b)?
  - What is the distribution of the coefficients *a* and *b*?
- **How Not to Hash** into Class Groups?
  - SageMath's hash function (inappropriate for crypto applications).
  - An attack for a VDF application using this broken hash function.
- **How to Hash** into Class Groups?
  - A simple algebraic hash function.
  - Our extended hash function of Wesolowski's hash function.
- **Future directions**, Open Questions

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What is a Class Group of Imaginary Quadratic Fields?

What is a class group? I.

- A binary quadratic form is a function $f(X,Y)=aX^2+bXY+cY^2$.
- The *discriminant* **Δ** of a binary quadratic form *f* is $\Delta(f):=b^2-4ac$.
- Let us fix a prime **Δ**.
  - For instance, **Δ=-23**.
- Cl(**Δ**) is the set of *integer* triplets *(a,b,c)* s.t. $\Delta=b^2-4ac$.
  - Cl(**-23**)={(1,1,6),(2,1,3),(2,-1,3)}. Easy to see: $a,b\leq\Delta^{1/2}/3$.
- Typically we **represent group elements** as *(a,b)*. No need for *c*.
  - One can further compress group elements, see [DGS21].
- There is a group structure on Cl(**Δ**) due to Gauss (1801).
- Dirichlet (1839): $|Cl(\Delta)|\approx O(|\Delta|^{1/2})$.
- **No efficient algorithm** to compute the order of Cl(**Δ**).

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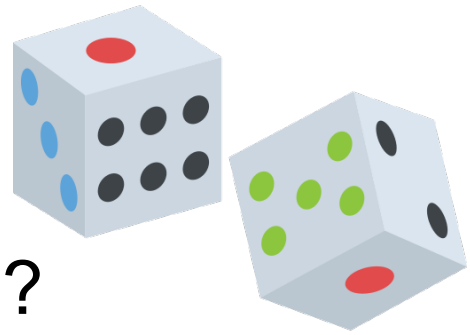
- *f* is called a reduced binary form iff.$-a<b\leq a$ and $a<c$.
- Binary quadratic forms by **linear transformations** are equivalent.
- More precisely, Cl(**Δ**) is the **set of reduced** binary quadratic forms.
- Analogy: equivalent residue classes mod **N**. 3≡11 mod **8**.

Desiderata for hashing to class groups

- Cl(**Δ**): the set of *reduced* binary quadratic forms *(a,b,c)* s.t. **Δ**=b^2-4ac.
  - Cl(**-47**)={(1,1,12),(2,1,6),(2,-1,6),(3,-1,4),(3,1,4)}.
- (Recall a,b≤$\mathbf{\Delta}^{1/2}$/3)
- What do we want from a hash-to-class group function **H**?
  - *Deterministic*: **H** is a deterministic function.
  - *Efficient*: computing **H** can be done fast.
  - *Collision resistance*: finding m_1,m_2 such that **H**(m_1)=**H**(m_2) is hard.
  - *One-way function*: given **H**(m_1), it is hard to compute m_1.
  - *Uniformly distributed*: the codomain of **H** is uniform.



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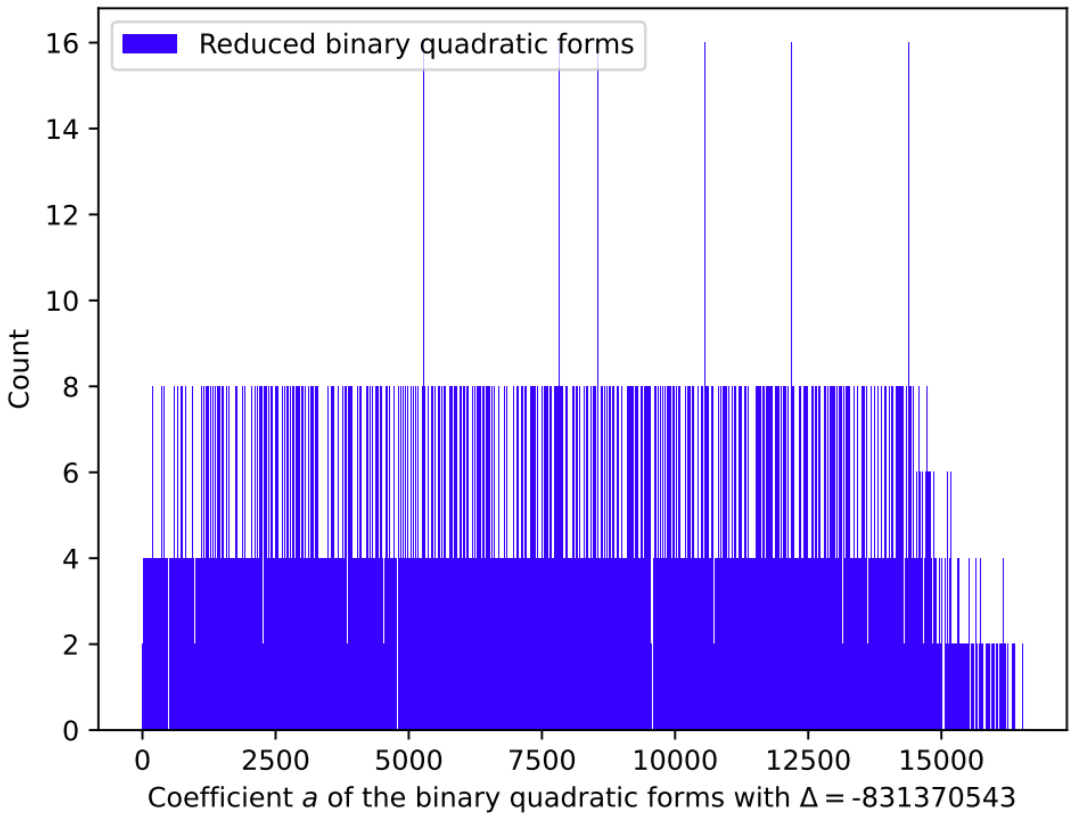
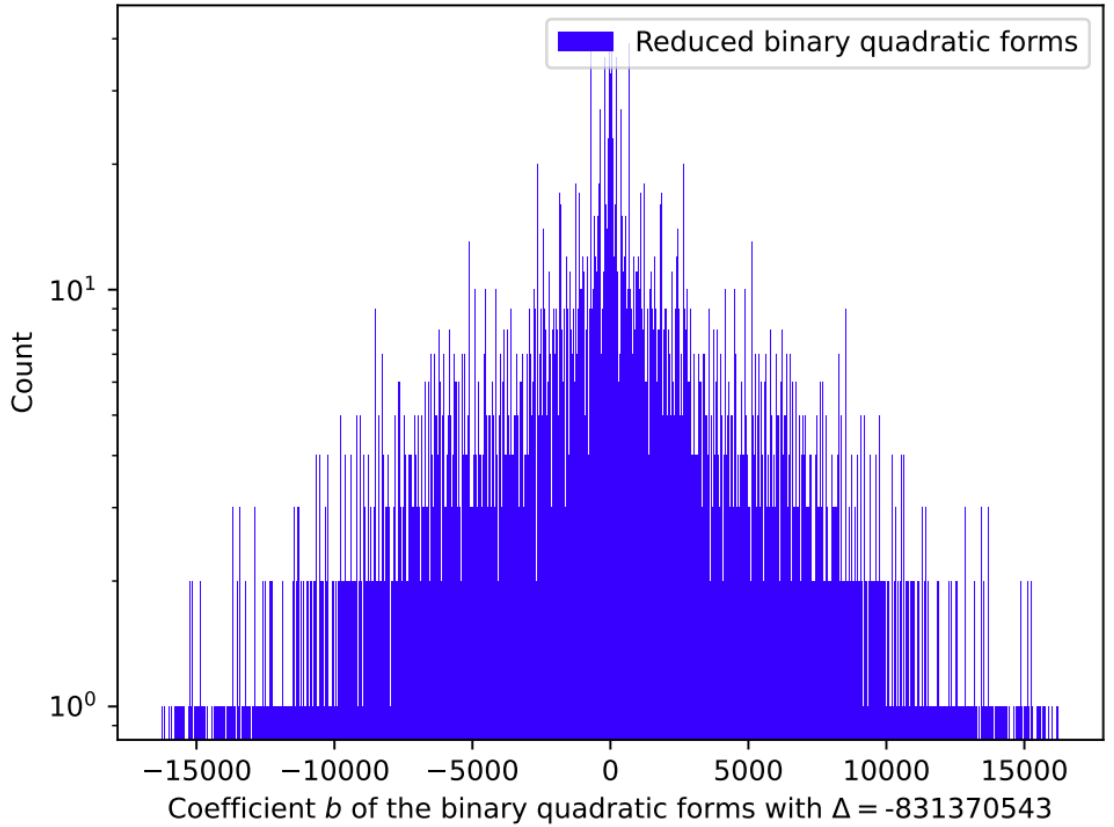
How Not to Hash into Class Groups of Imaginary Quadratic Fields?

Distribution of (a,b) in Cl(-831370543)

Distribution of coefficient “a”



Distribution of coefficient “b”



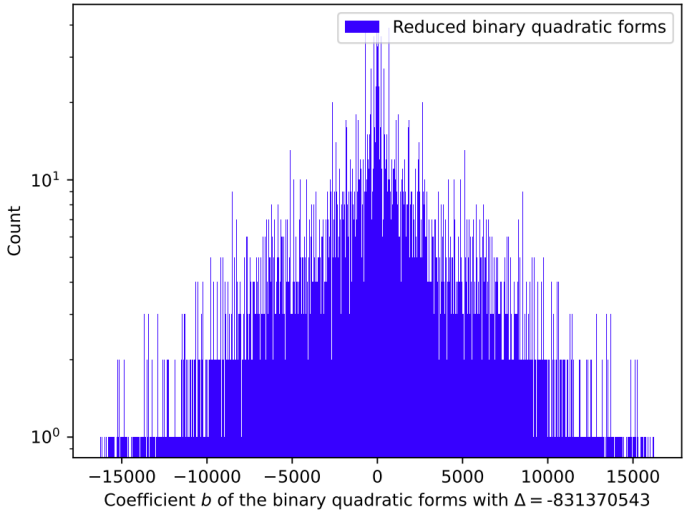
A conjecture for number theorists

Conjecture (The distribution of "b" in Cl(Δ))

- For a fixed **Δ**, there exist constants c_1, c_2 such that for any large enough *B*, the average number of forms in Cl(**Δ**) having a coefficient *b* in [B,2B) follows asymptotically

$$c_1 \log(|\mathbf{\Delta}|) - c_2 \log(b) + O(1),$$

whenever $\mathbf{\Delta} \to -\infty$.



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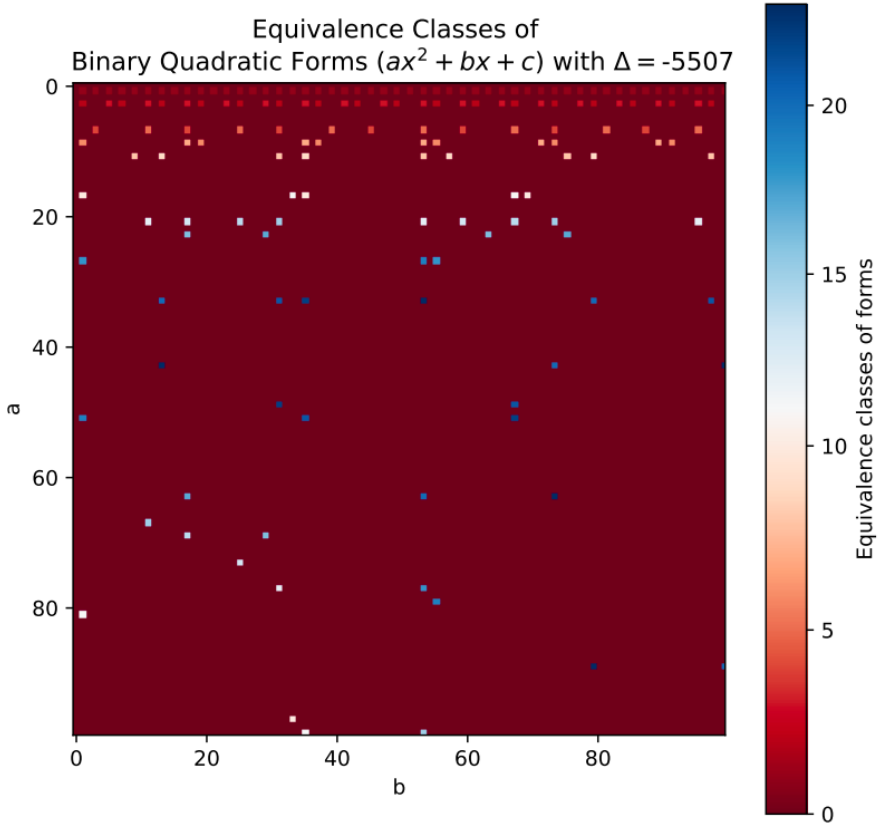
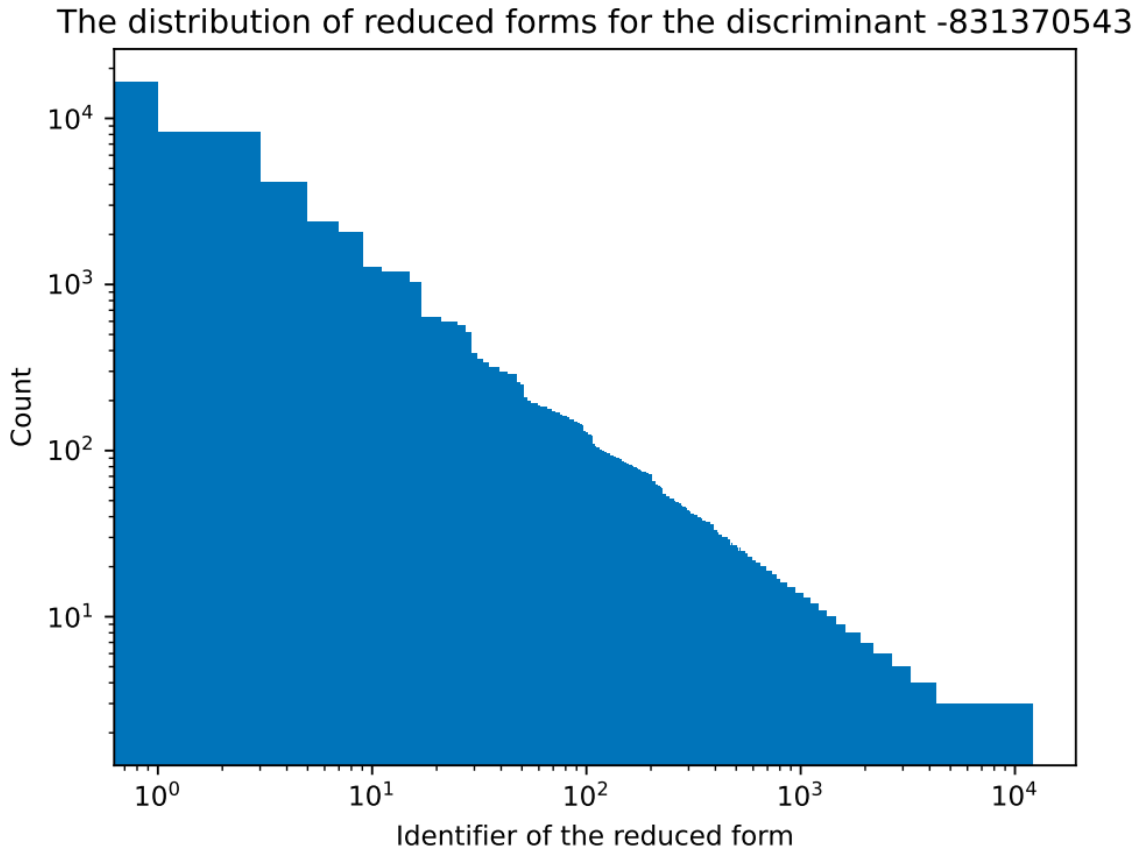
How (Not) to Hash into Class Groups? I.

1. Sampling a random coefficient “*b*”?
   a. Non-uniform distribution.
2. Sampling a random coefficient “*a*”?
   a. Non-uniform distribution
3. For an input message *m*, let **H**(m):=f^m.
   a. In many applications, unknown DLOG is necessary.

How (Not) to Hash into Class Groups? II.

- Random form and reduce? *(Implemented by SageMath)*





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How to Hash into Class Groups of Imaginary Quadratic Fields?

How to Hash into Class Groups? I.

How to hash into class groups? Construction #1.

Public parameters: $\mathbb{G} \xleftarrow{R} GGen(\lambda), \mathsf{H}_0 : \{0,1\}^* \to \{0,1\}^d$ a secure hash function. A precomputation outputs $\{f_i\}_{i=1}^d \in \mathsf{Cl}(\Delta)^d$ forms with small prime norms, i.e., $\forall i \in [1,d] : f_i = (p_i, b_i) \in \mathsf{Cl}(\Delta)$ for some "small" prime $p_i \in [2, \mathcal{O}(\log|\Delta|)]$.
Inputs: $x \in \{0,1\}^*$.

1. Let $b := \mathsf{H}_0(x)$ and b_i be the ith bit of b.
2. Return: $H(x) := \prod_{i=1}^{d} f_i^{b_i} (\in \mathsf{Cl}(\Delta))$.

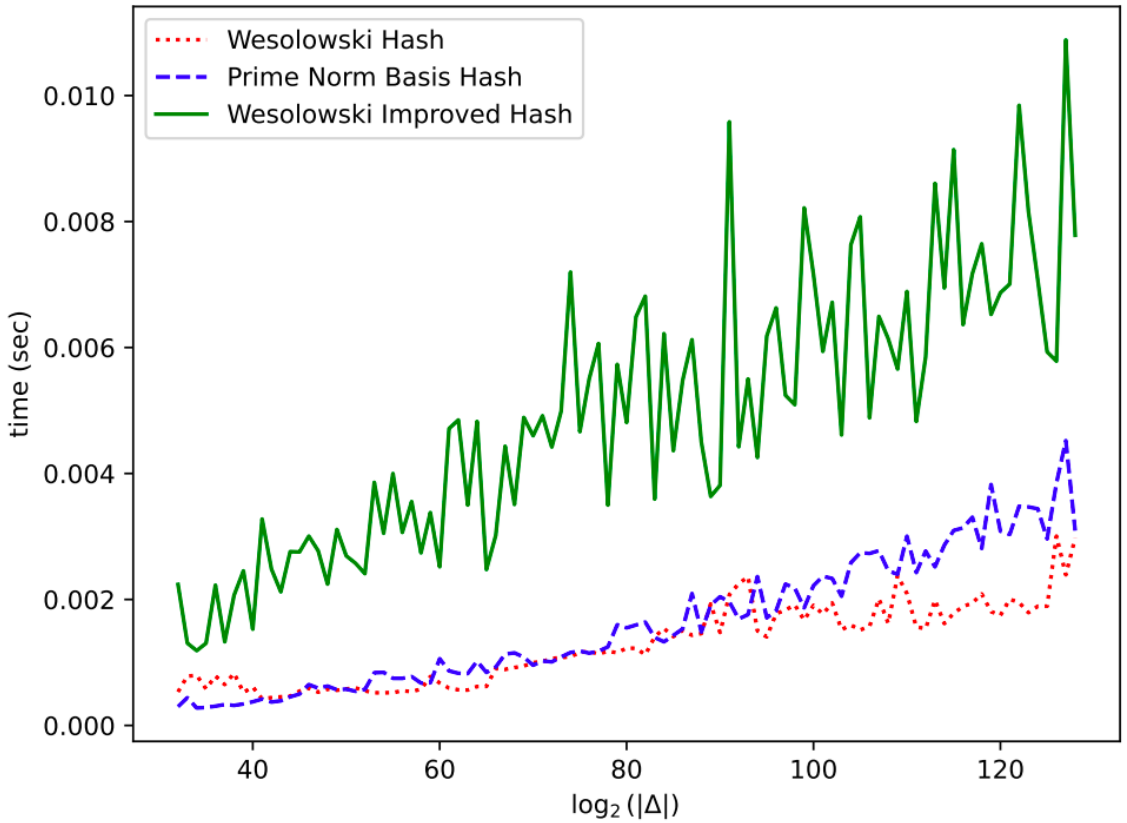
How to Hash into Class Groups? II.

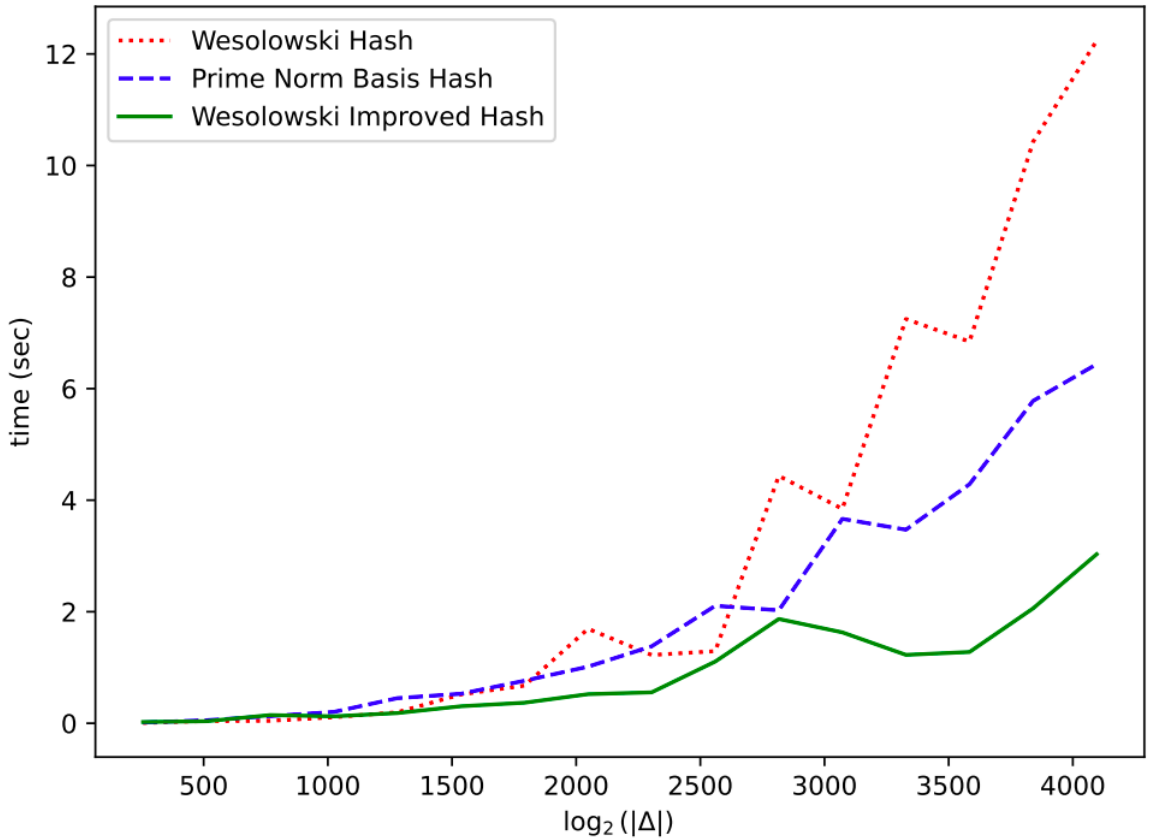
How to hash into class groups? Construction #2.

Public parameters: $\mathbb{G} \xleftarrow{R} GGen(\lambda), \mathsf{H}_{\mathsf{Primes}} : \{0,1\}^* \to \mathsf{Primes}(\lambda)$ a cryptographically secure hash function. Let Δ be the discriminant corresponding to the generated group $\mathbb{G}$.
Inputs: $x \in \{0,1\}^*$.

1. $i = 0$.
2. Compute $p := \mathsf{H}_{\mathsf{Primes}}(x||i)$. // For the sake of uniform distribution p must be large, i.e., $p \xleftarrow{\$} [0, \sqrt{|\Delta|/4}]$. We also need to ensure the reducedness of the form.
3. If $\left(\frac{\Delta}{p}\right) = 1$ then let $b := (\sqrt{\Delta} \bmod p)$.
   - else $i = i + 1$, and *goto* 2. //There is no prime form $(p, \cdot) \in \mathsf{Cl}(\Delta)$, cf. Theorem 1.
4. Let $s := (p \mod 3)$. // If $\left(\frac{\Delta}{p}\right) = 1$, then there are two modular square roots: b and $-b$. We need to choose one of them deterministically.
   - If $s == 0 \vee s == 1$: Return $H(x) := (p, b)$.
   - elif $s == 2$: Return $H(x) := (p, -b)$.

Performance Evaluation





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Open Problems and Future Research Directions

Future Work and Research Directions

- More efficient hash-to-class group functions?
  - More efficient and constant time *hash-to-prime functions*?
  - Efficiency in an arithmetic circuit computation model, e.g., ZKPs, MPC.
- **Constant time** hash-to-class group functions?
- **More applications** of hash-to-class group functions?
- More performant class group cryptography libraries are needed!
  - Class group cryptography implemented for ZKP circuits! Pls help!
    - Noir, Cairo, Circom.

"Apply" Slide

- Is there any protocol or application in my firm where I need to sample a random binary quadratic form?
- If we apply a class group verifiable delay function, then likely we need a secure hash-to-class group function!
- Lots of room for efficiency improvement: *class groups are still not ready for most (resource-constrained) applications*.
- The world needs more class group crypto implementation! ***Pls help***!
  - Class group arithmetic implemented for smart contracts, blockchains.
  - Class group arithmetic implemented for zero-knowledge circuits.

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Thank you very much!

Q&A

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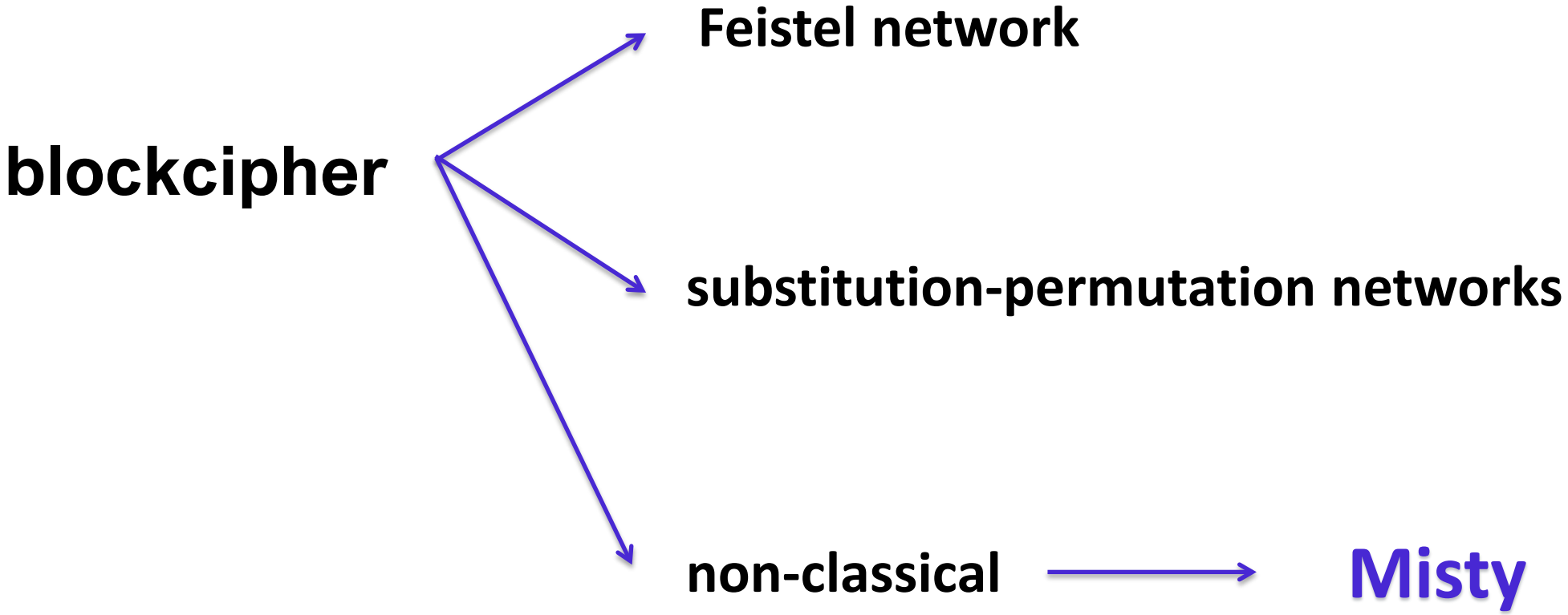
One Community.

SESSION ID: CRYP-M05B

Sequential Indifferentiability of 7-round Misty Structures

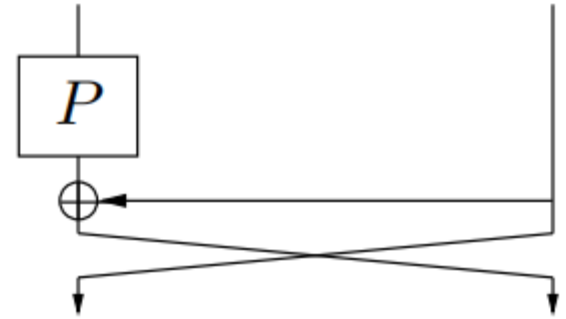
Jiayi Ai

Blockcipher

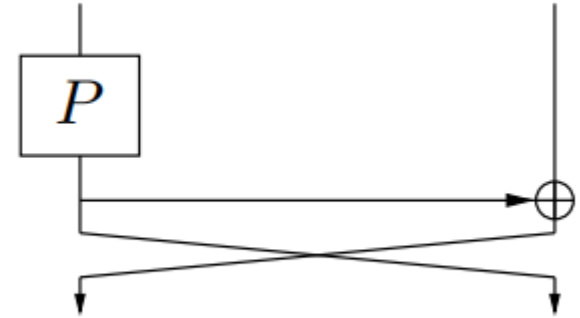


Structure of a Misty work

Misty → **misty-L**, **misty-R**

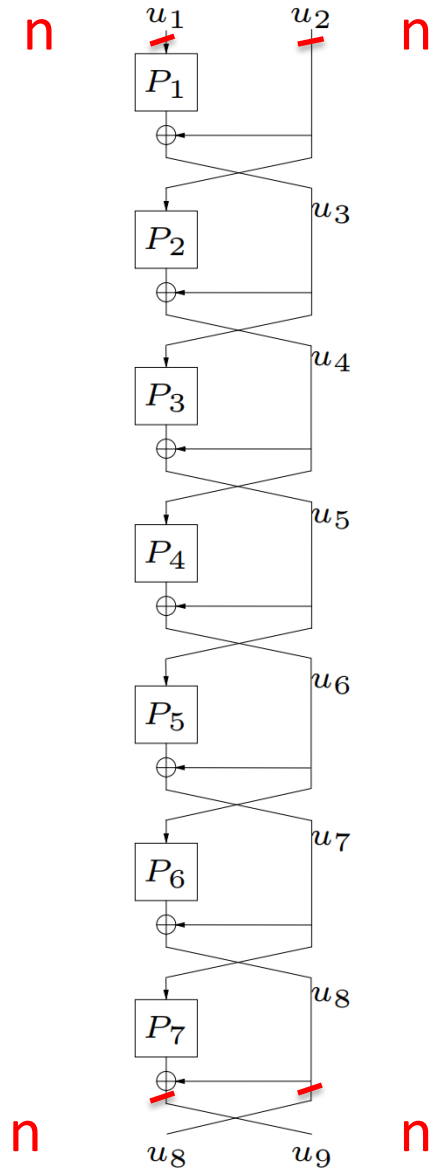


$$\text{Misty-L}^{P}(L, R) := \big(R, P(L) \oplus R\big)$$



$$\text{Misty-R}^{P}(L, R) := \big(P(L) \oplus R, P(L)\big)$$

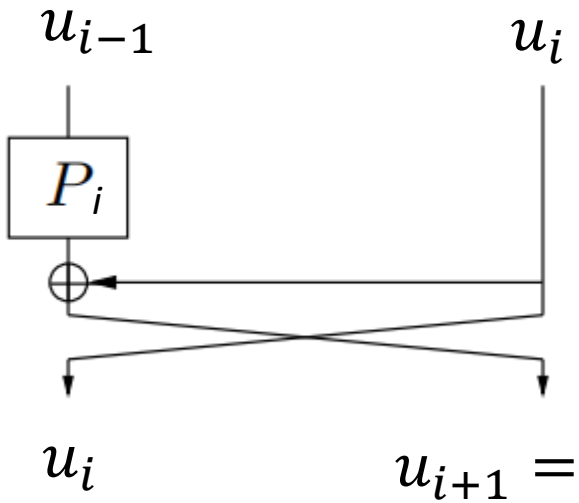
Structure of a Misty work



$$\text{random permutation } P : \{0,1\}^n \rightarrow \{0,1\}^n$$

Each round:

round permutation



$$u_{i+1} = P(u_{i-1}) \oplus u_i$$

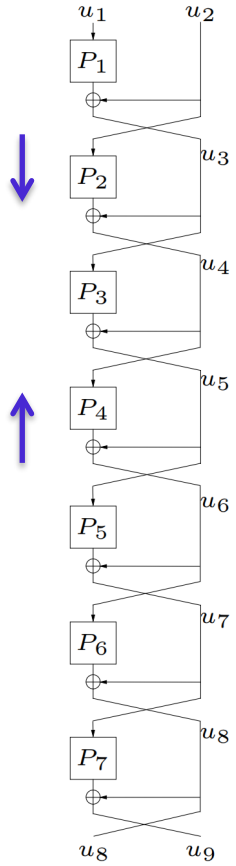
Indistinguishability

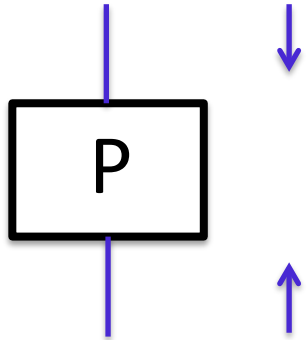
Indifferentiability

D

real world

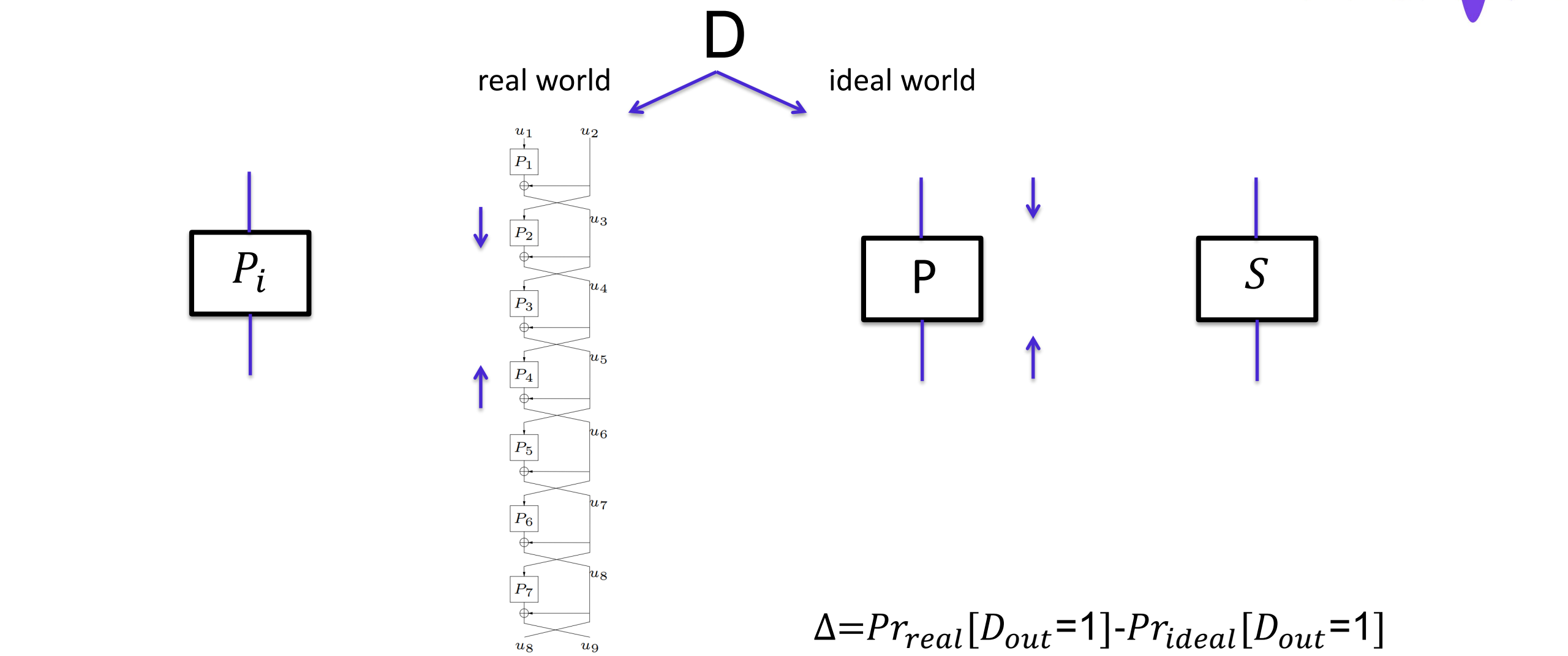
ideal world





$$\Delta = Pr_{real}[D_{out}=1] - Pr_{ideal}[D_{out}=1]$$

Indifferentiability[MRH04]



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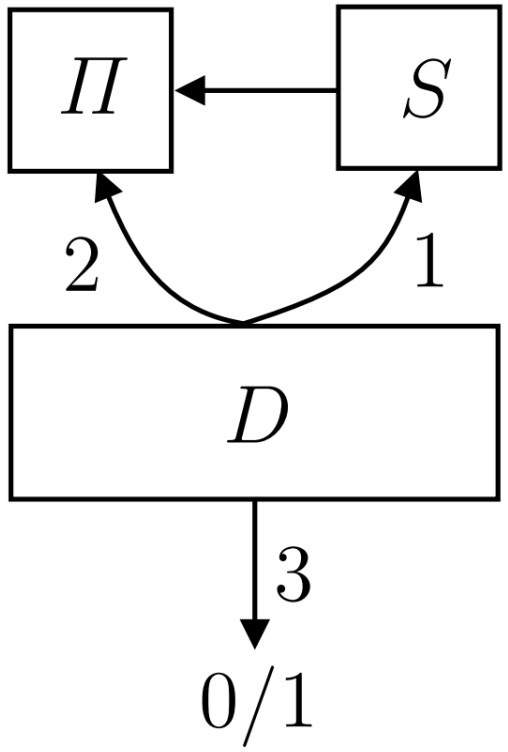
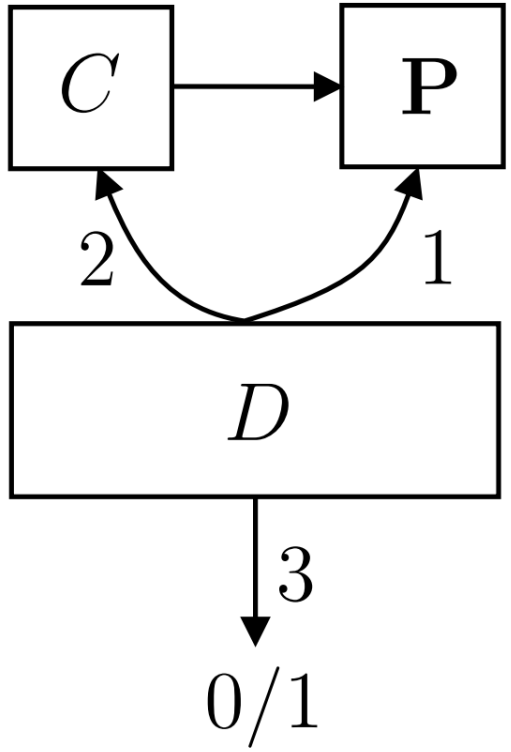
Sequential Indifferentiability

$\{0,1\}^{2n} \to \{0,1\}^{2n}$





$\{0,1\}^{n} \to \{0,1\}^{n}$

Simulator

$\prod_i$ a mapping of entries $(\delta, w) \in \{+, -\} \times \{0,1\}^n$ to values $w' \in \{0,1\}^n$

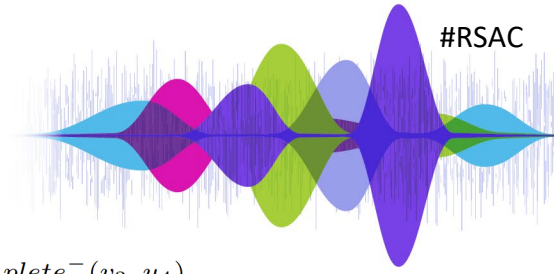
i = 1,...,7

$\text{Query}(i, \delta, w)$ i = 1,...,7

$\delta \in \{+, -\}$ denotes whether the query is direct or inverse

$w \in \{0,1\}^n$ indicates the value being queried

Simulator



1: **Simulator** $S^{\mathbf{\Pi},\mathbf{P}}$
2: **Variables:** tables Π_1,...,Π_7, all initially empty
3: **public procedure** Query(i, δ, w)
4: **if** $(\delta, w) \notin \Pi_i$ **then**
5: $w' := P_i(\delta, w)$
6: $\Pi_i(\delta, w) := w'$
7: $\Pi_i(\overline{\delta}, w') := w$
8: **if** $i = 3$ **then**
9: **if** $\delta = +$ **then**
10: $v_3 = w'$
11: **forall** $u_4 \in \Pi_4^+$ **do**
12: $Complete^+(v_3, u_4)$
13: **else if** $\delta = -$ **then**
14: $v_3 = w$
15: **forall** $u_4 \in \Pi_4^+$ **do**
16: $Complete^-(v_3, u_4)$
17: **else if** $i = 4$ **then**
18: **if** $\delta = +$ **then**
19: $u_4 = w$
20: **forall** $v_3 \in \Pi_3^-$ **do**
21: $Complete^+(v_3, u_4)$
22: **else if** $\delta = -$ **then**
23: $u_4 = w'$
24: **forall** $v_3 \in \Pi_3^-$ **do**
25: $Complete^-(v_3, u_4)$

26: **private procedure** $Complete^+(v_3, u_4)$
27: $v_4 := \Pi_4(+, u_4)$
28: $u_5 := v_3 \oplus u_4$
29: $v_5 := \text{Query}(5, +, u_5)$
30: $u_6 := v_4 \oplus u_5$
31: $u_7 := v_5 \oplus u_6$
32: $u_3 := \Pi_3(-, v_3)$
33: $v_2 := u_3 \oplus u_4$
34: $u_2 := \text{Query}(2, -, v_2)$
35: $v_1 := u_2 \oplus u_3$
36: $u_1 := \text{Query}(1, -, v_1)$
37: $(u_8, u_9) := \mathbf{\Pi}(+, (u_1, u_2))$
38: $v_6 := u_7 \oplus u_8$
39: $v_7 := u_8 \oplus u_9$
40: // Adapt: may overwrite
41: $\Pi_6(+, u_6) := v_6$
42: $\Pi_6(-, v_6) := u_6$
43: $\Pi_7(+, u_7) := v_7$
44: $\Pi_7(-, v_7) := u_7$

45: **private procedure** $Complete^-(v_3, u_4)$
46: $u_3 := \Pi_3(-, v_3)$
47: $v_2 := u_3 \oplus u_4$
48: $v_4 := \Pi_4(+, u_4)$
49: $u_5 := v_3 \oplus u_4$
50: $v_5 := \text{Query}(5, +, u_5)$
51: $u_6 := v_4 \oplus u_5$
52: $v_6 := \text{Query}(6, +, u_6)$
53: $u_7 := v_5 \oplus u_6$
54: $v_7 := \text{Query}(7, +, u_7)$
55: $u_8 := v_6 \oplus u_7$
56: $u_9 := v_7 \oplus u_8$
57: $(u_1, u_2) := \mathbf{\Pi}(-, (u_8, u_9))$
58: $v_1 := u_2 \oplus u_3$
59: // Adapt: may overwrite
60: $\Pi_1(+, u_1) := v_1$
61: $\Pi_1(-, v_1) := u_1$
62: $\Pi_2(+, u_2) := v_2$
63: $\Pi_2(-, v_2) := u_2$

Simulator

Simulator $S^{\mathbf{\Pi},\mathbf{P}}$
Variables: tables Π_1,...,Π_7, all initially empty
public procedure Query(i, δ, w)
  if $(\delta, w) \notin \Pi_i$ **then**
    $w' := P_i(\delta, w)$
    $\Pi_i(\delta, w) := w'$
    $\Pi_i(\bar{\delta}, w') := w$
    if $i = 3$ **then**
      if $\delta = +$ **then**
        $v_3 = w'$
        forall $u_4 \in \Pi_4^+$ **do**
          $Complete^+(v_3, u_4)$
      else if $\delta = -$ **then**
        $v_3 = w$
        forall $u_4 \in \Pi_4^+$ **do**
          $Complete^-(v_3, u_4)$
    else if $i = 4$ **then**
      if $\delta = +$ **then**
        $u_4 = w$
        forall $v_3 \in \Pi_3^-$ **do**
          $Complete^+(v_3, u_4)$
      else if $\delta = -$ **then**
        $u_4 = w'$
        forall $v_3 \in \Pi_3^-$ **do**
          $Complete^-(v_3, u_4)$

private procedure $Complete^+(v_3, u_4)$
  $v_4 := \Pi_4(+, u_4)$
  $u_5 := v_3 \oplus u_4$
  $v_5 := \text{Query}(5, +, u_5)$
  $u_6 := v_4 \oplus u_5$
  $u_7 := v_5 \oplus u_6$
  $u_3 := \Pi_3(-, v_3)$
  $v_2 := u_3 \oplus u_4$
  $u_2 := \text{Query}(2, -, v_2)$
  $v_1 := u_2 \oplus u_3$
  $u_1 := \text{Query}(1, -, v_1)$
  $(u_8, u_9) := \mathbf{\Pi}(+, (u_1, u_2))$
  $v_6 := u_7 \oplus u_8$
  $v_7 := u_8 \oplus u_9$
  // Adapt: may overwrite
  $\Pi_6(+, u_6) := v_6$
  $\Pi_6(-, v_6) := u_6$
  $\Pi_7(+, u_7) := v_7$
  $\Pi_7(-, v_7) := u_7$

private procedure $Complete^-(v_3, u_4)$
  $u_3 := \Pi_3(-, v_3)$
  $v_2 := u_3 \oplus u_4$
  $v_4 := \Pi_4(+, u_4)$
  $u_5 := v_3 \oplus u_4$
  $v_5 := \text{Query}(5, +, u_5)$
  $u_6 := v_4 \oplus u_5$
  $v_6 := \text{Query}(6, +, u_6)$
  $u_7 := v_5 \oplus u_6$
  $v_7 := \text{Query}(7, +, u_7)$
  $u_8 := v_6 \oplus u_7$
  $u_9 := v_7 \oplus u_8$
  $(u_1, u_2) := \mathbf{\Pi}(-, (u_8, u_9))$
  $v_1 := u_2 \oplus u_3$
  // Adapt: may overwrite
  $\Pi_1(+, u_1) := v_1$
  $\Pi_1(-, v_1) := u_1$
  $\Pi_2(+, u_2) := v_2$
  $\Pi_2(-, v_2) := u_2$

Simulator

Simulator $S^{\mathbf{\Pi},\mathbf{P}}$
Variables: tables Π_1,...,Π_7, all initially empty
public procedure Query(i, δ, w)
  if $(\delta, w) \notin \Pi_i$ **then**
    $w' := P_i(\delta, w)$
    $\Pi_i(\delta, w) := w'$
    $\Pi_i(\bar{\delta}, w') := w$
    if $i = 3$ **then**
      if $\delta = +$ **then**
        $v_3 = w'$
        forall $u_4 \in \Pi_4^+$ **do**
          $Complete^+(v_3, u_4)$
      else if $\delta = -$ **then**
        $v_3 = w$
        forall $u_4 \in \Pi_4^+$ **do**
          $Complete^-(v_3, u_4)$
    else if $i = 4$ **then**
      if $\delta = +$ **then**
        $u_4 = w$
        forall $v_3 \in \Pi_3^-$ **do**
          $Complete^+(v_3, u_4)$
      else if $\delta = -$ **then**
        $u_4 = w'$
        forall $v_3 \in \Pi_3^-$ **do**
          $Complete^-(v_3, u_4)$

private procedure $Complete^+(v_3, u_4)$
  $v_4 := \Pi_4(+, u_4)$
  $u_5 := v_3 \oplus u_4$
  $v_5 := \text{Query}(5, +, u_5)$
  $u_6 := v_4 \oplus u_5$
  $u_7 := v_5 \oplus u_6$
  $u_3 := \Pi_3(-, v_3)$
  $v_2 := u_3 \oplus u_4$
  $u_2 := \text{Query}(2, -, v_2)$
  $v_1 := u_2 \oplus u_3$
  $u_1 := \text{Query}(1, -, v_1)$
  $(u_8, u_9) := \mathbf{\Pi}(+, (u_1, u_2))$
  $v_6 := u_7 \oplus u_8$
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  // Adapt: may overwrite
  $\Pi_6(+, u_6) := v_6$
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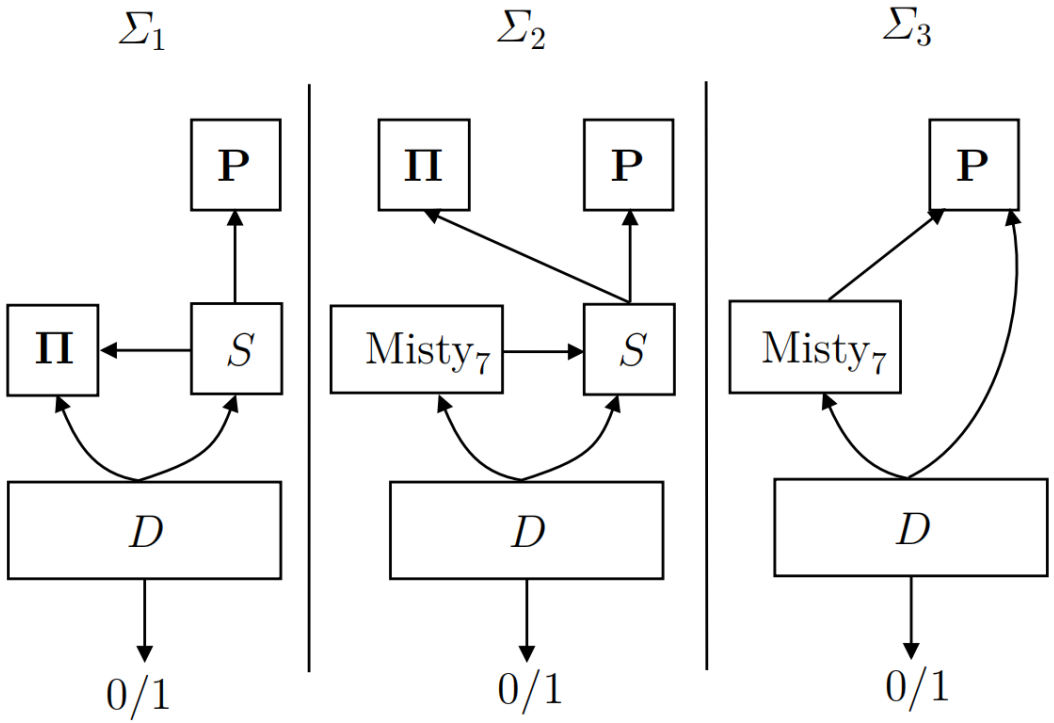
  $\Pi_7(+, u_7) := v_7$
  $\Pi_7(-, v_7) := u_7$

private procedure $Complete^-(v_3, u_4)$
  $u_3 := \Pi_3(-, v_3)$
  $v_2 := u_3 \oplus u_4$
  $v_4 := \Pi_4(+, u_4)$
  $u_5 := v_3 \oplus u_4$
  $v_5 := \text{Query}(5, +, u_5)$
  $u_6 := v_4 \oplus u_5$
  $v_6 := \text{Query}(6, +, u_6)$
  $u_7 := v_5 \oplus u_6$
  $v_7 := \text{Query}(7, +, u_7)$
  $u_8 := v_6 \oplus u_7$
  $u_9 := v_7 \oplus u_8$
  $(u_1, u_2) := \mathbf{\Pi}(-, (u_8, u_9))$
  $v_1 := u_2 \oplus u_3$
  // Adapt: may overwrite
  $\Pi_1(+, u_1) := v_1$
  $\Pi_1(-, v_1) := u_1$
  $\Pi_2(+, u_2) := v_2$
  $\Pi_2(-, v_2) := u_2$

Intermediate System



$$\left|\Pr[D^{\Sigma_1(\mathbf{\Pi},S^{\mathbf{\Pi},P})}=1]-\Pr[D^{\Sigma_2(Misty^{S^{\mathbf{\Pi},P}},S^{\mathbf{\Pi},P})}=1]\right|\leq\frac{32q^4}{N}.$$

$$\left|\Pr\left[D^{\Sigma_2(Misty^{S^{\mathbf{\Pi},P}},S^{\mathbf{\Pi},P})}=1\right]-\Pr\left[D^{\Sigma_3(Misty^{P},\boldsymbol{P})}=1\right]\right|\leq\frac{153q^4}{N}.$$

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One Community.

Thank You